A Small Cosmological Constant
from Warped Compactification with Branes

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Abstract

We present a possible explanation for the smallness of the observed cosmological constant using a variant of the Randall-Sundrum(RS)-Goldberger-Wise paradigm for a warped extra dimension. In contrast to RS, we imagine that we are living on the positive tension Planck brane, or on a zero-tension TeV brane. In our model there are two solutions for the scalar field in the bulk and the corresponding brane separations, one of which is tuned to have zero cosmological constant. We show that in the other solution, which is a false vacuum state, the 4-D cosmological constant can be naturally small, due to exponential suppression by the warp factor. The radion is in the milli-eV mass range, and if we live on a TeV brane its couplings are large enough that it can measurably alter the gravitational force at submillimeter distances. In this case the Kaluza-Klein excitations of the graviton can also contribute to submillimeter deviations from Newtonian gravity, and we have in addition the collider phenomenology of the usual TeV-scale radion.

1. Introduction. There is mounting evidence that we live in a universe with a vacuum energy density Λ which is about 0.7 of the critical density [1]. This value is close to 120 orders of magnitude less than the Planck density, M_p^4, which would be the natural expectation for the size of Λ from quantum field theory. If there exists some mechanism to make Λ small, it would seem much simpler if its value was zero than $10^{-120} M_p^4$. Our motivation in this letter is to present a possible explanation for this enormous hierarchy of scales, taking advantage of recent ideas involving 3-branes embedded in an extra dimension.

A similar hierarchy problem, though much less severe, is that of the weak scale (100 GeV) versus the (reduced) Planck scale, $M_p = (8\pi G_N)^{-1/2} = 2.43 \times 10^{18}$ GeV. Randall and Sundrum (RS) presented an interesting way of explaining this small ratio using two 3-branes in a 5-D anti-deSitter space with a compact extra dimension [2]-[3]. It is tempting to look for a similar application of the RS idea for the cosmological constant problem; several attempts to use one or more extra dimensions to get a small Λ have recently been made [4]-[5]. In this paper we will suggest that the RS idea can, with only minor changes, account for the smallness of the observed Λ. Our model relies heavily on stabilization of the distance between two branes using a bulk scalar field, as was suggested by Goldberger and Wise [6].

Our explanation only partially addresses the cosmological constant problem, in that we assume there is some unknown mechanism which forces the ground state of the universe to have vanishing 4-D vacuum energy. We show that there can be a false vacuum state with Λ nonzero and exponentially small, as illustrated in figure 1. Its size is determined by the separation between the two branes, which in turn depends on the potential of the bulk scalar field ϕ. The degree of freedom which is varying between the true and false vacua is the *radion*, the dynamical field associated with fluctuations in the size of the extra dimension.

To set the stage, we will be working in a 5-D theory with the extra-dimensional coordinate r, whose metric is parametrized by

$$ds^2 = e^{2A(r)} \left(dt^2 - e^{2\sqrt{\bar{\Lambda}}t} \sum_i dx_i^2 \right) - dr^2. \tag{1}$$

We note that slices of constant r represent 4-D deSitter spaces with vacuum energy $\Lambda = 3\bar{\Lambda}/(8\pi G)$, where G is the 4-D Newton constant. There are two 3-branes, located $r = 0$ and $r = r_1$ respectively. There is also a scalar field ϕ with potential $V(\phi)$ in the bulk, and separate potentials $\lambda_0(\phi)$ and $\lambda_1(\phi)$ on the two branes. $V(\phi)$ contains a negative bulk cosmological constant which induces the behavior $A(r) \propto -r$, so that $e^{2A(r)}$ is exponentially decreasing. Using notation similar to that of reference [7], the action is

$$S = \int d^5x \sqrt{g} \left(-\frac{1}{4\kappa^2} R + \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \lambda_0(\phi)\delta(r) - \lambda_1(\phi)\delta(r - r_1) \right) \tag{2}$$

Here $\kappa^2 = \frac{1}{2}M^{-3}$ defines the 5-D gravity scale M, which is assumed to be of order M_p. To simplify the analysis, we will take the brane potentials to have the form

$$\lambda_i(\phi) = T_i + \gamma_i(\phi^2 - \phi_i^2)^2 \tag{3}$$

with $\gamma_i \to \infty$, *i.e.,* the brane potentials are stiff. This will make the boundary conditions for the scalar field simply

$$\phi(0) = \phi_0, \qquad \phi(r_1) = \phi_1, \tag{4}$$

and the potentials will have the values $\lambda_i(\phi_i) = T_i$, where T_i denotes the tension of the respective brane.

In the remainder we will show that for the same simple choice of bulk potential used in the exact solution of ref. [7], it is possible to find a metastable solution to the equations of motion whose false vacuum energy (in the 4-D effective theory) is exponentially close to that of the true vacuum. By assuming the latter has vanishing cosmological constant, we can thus explain the small observed value of Λ in our universe by imagining that we are stuck in the false vacuum.

2. Superpotential method. To construct solutions to the coupled equations for 5-D gravity and the scalar field, we will take advantage of the superpotential method discussed in ref. [7]. One introduces a superpotential $W(r)$ and a function

$$\gamma(r) = \sqrt{1 + \frac{9\bar{\Lambda}}{\kappa^4 W(r)^2} e^{-2A(r)}} \tag{5}$$

such that the desired potential $V(\phi)$ is given by

$$V(\phi(r)) = \frac{1}{8\gamma^2} \left(\frac{\partial W}{\partial \phi}\right)^2 - \frac{\kappa^2}{3} W^2. \tag{6}$$

Then solutions to the coupled equations for A and ϕ can be generated from the first order equations

$$A' = -\frac{\kappa^2}{3} W\gamma; \qquad \phi' = \frac{1}{2\gamma} \frac{\partial W}{\partial \phi}. \tag{7}$$

For flat branes (when $\Lambda = 0$), W can be regarded as a function of ϕ alone, but for bent branes ($\Lambda \neq 0$), it must be considered as a function of r, and in that case one interprets $\frac{\partial W}{\partial \phi} = \frac{1}{\phi'} \frac{dW}{dr}$. The boundary conditions at the brane positions $r = 0$ and $r = r_1$ are

$$\begin{align} \lambda_0(\phi_0) &= W\gamma|_{r=0} ; & \phi(0) &= \pm\phi_0 \tag{8} \\ \lambda_1(\phi_1) &= -W\gamma|_{r=r_1}; & \phi(r_1) &= \pm\phi_1 \tag{9} \end{align}$$

3. Solutions with vanishing Λ. Let us first present a ground state solution in which the branes are flat and the 4-D cosmological constant Λ is tuned to be zero, hence $\gamma(r) = 1$. We will later perturb this solution with a small positive value of Λ. The unperturbed functional form for the superpotential is taken to be

$$W_0(\phi) = \frac{3}{\kappa^2 L} - b\phi^2, \tag{10}$$

where L is a length scale that turns out to be the curvature radius of the 5-D anti-deSitter space in the limit where the back reaction of the scalar on the geometry is small. The

resulting scalar potential is simply a polynomial in ϕ of degree 4. It is easy to integrate the equations of motion (7) to obtain the solutions [7]

$$
\begin{aligned}
\phi &= \phi_0 \, e^{-br} \\
A &= \hat{A} - \frac{r}{L} - \frac{\kappa^2}{6}\phi_0^2 \, e^{-2br},
\end{aligned}
\tag{11}
$$

where $\hat{A}$ is a constant of integration (we have already chosen ϕ_0 so as to satisfy the boundary condition on ϕ at $r = 0$.) The value of $\hat{A}$ is physically irrelevant, so we can for convenience choose it such that $A(0) = 0$. Applying the boundary condition on ϕ at the second brane determines r_1, the size of the extra dimension:

$$
e^{-br_1} = \frac{\phi_1}{\phi_0}
\tag{12}
$$

This solution is essentially the same as the ones found in references [6] and in particular [7], differing from the latter only by our choice of stiff brane potentials. The stability of this solution against small perturbations has been demonstrated in [8]. The large hierarchy of energy scales between the two branes is generated by assuming that b is parametrically small compared to $1/L$. In this case, although $e^{-br_1} \lesssim 1$, the warp factor will be exponentially small, $e^{-r_1/L} \ll 1$.

The remaining boundary conditions are satisfied by the constraints

$$
\begin{aligned}
T_0 &= \frac{3}{\kappa^2 L} - b\phi_0^2 \\
T_1 &= -\frac{3}{\kappa^2 L} + b\phi_1^2
\end{aligned}
$$

$$
\tag{13}
$$
$$
\tag{14}
$$

on the brane tensions. The fact that we have two fine-tunings instead of the one which is expected on the basis of counting parameters [7] is due to our choice of superpotential W_0. There is actually a family of superpotentials giving rise to the same physical potential, and these can be found by integrating the differential equation (6). A new constant of integration would then arise, which we have already fixed by our choice of W_0. If the brane tensions were not related to each other as prescribed by (13-14) we would be forced to find the appropriate form of W_0. Thus there is only one essential fine-tuning, and this is the one that corresponds to setting the 4-D cosmological constant Λ to zero.

It is important to notice that, in addition to the solution at finite brane separation, there is another one with the second brane at $r = \infty$. In other words, the second brane is simply removed from the picture. The extra boundary conditions associated with the second brane are no longer relevant when it is at infinity since (due to the vanishing warp factor) it makes no contribution to the action.

4. Nearby solutions with nonzero Λ. Let us suppose that the tuning of brane tension T_0, eq. (13), is enforced, so that the solution where the second brane removed still exists, with vanishing 4-D cosmological constant. Consider what happens to the other solution if

the tension T_1 is no longer tuned according to condition (14). We expect that Λ is no longer zero in this case. The equations of motion with nonzero Λ are difficult to solve exactly, so we will instead solve them perturbatively, to first order in Λ.

The first step in this procedure is to realize that, although $V(\phi)$ should still have the same functional form as in the $\Lambda = 0$ solutions, the superpotential need not be the same. In general, it must be corrected in such a way that $V(\phi)$ is unchanged. Let us denote the new solutions and superpotential by

$$\phi_\Lambda \cong \phi + \delta\phi; \qquad A_\Lambda \cong A + \delta A$$
$$W_\Lambda(r) \cong W(\phi_\Lambda(r)) + \delta W(r) \tag{15}$$

where the quantities $\phi(r)$ and $A(r)$ refer to the bulk solutions with $\Lambda = 0$ found in the previous section, and $W(\phi)$ is the corresponding superpotential (10). By taking the variation of eq. (6) and keeping terms which are first order in Λ we get a differential equation for δW,

$$\delta W' - \frac{4}{3}\kappa^2 W \delta W = \frac{9}{4\kappa^4}\bar{\Lambda}e^{-2A}\left(\frac{1}{W}\frac{\partial W}{\partial\phi}\right)^2 \tag{16}$$

which by using the equations of motions (7) can be written as

$$(e^{4A}\delta W)' = \bar{\Lambda}e^{2A}\left(\frac{\phi'}{A'}\right)^2. \tag{17}$$

This can be formally integrated to solve for $\delta W(r)$,

$$\delta W = \bar{\Lambda}e^{-4A}\int_0^r e^{2A}\left(\frac{\phi'}{A'}\right)^2 dr + Ce^{-4A} \tag{18}$$

To determine the constant of integration C, we should impose the boundary condition (8) on the tension of the first brane. Recall that the value T_0 was already fixed by demanding the existence of the solution with only one brane and $\Lambda = 0$. Since the brane potentials are assumed to be stiff, $\lambda_0(\phi_0) = T_0$ regardless of whether $\Lambda = 0$ or not. This leads to the requirement that the variation of eq. (8) vanishes:

$$\delta W(0) + \frac{9\bar{\Lambda}e^{-2A(0)}}{2\kappa^4 W(\phi_0)} = 0. \tag{19}$$

Recalling the definition $A(0) = 0$, this fixes C to be

$$C = -\frac{9\bar{\Lambda}}{2\kappa^4 W(\phi_0)} \tag{20}$$

In fact, because of our assumption that $b \ll 1/L$, needed to get a large hierarchy of scales, the C term is the dominant one in δW. The first term in (18) is of order $\phi'^2 \sim b^2$ and can therefore be neglected.

Once δW is known, it is straightforward to find the perturbations $\delta\phi$ and δA. In particular, the equation for $\delta\phi$ is

$$\delta\phi' = \frac{\delta W'}{2\phi'} - \frac{9\bar{\Lambda}e^{-2A}}{2\kappa^4 W(\phi)^2}\phi' + \frac{1}{2}\frac{\partial^2 W}{\partial\phi^2}\delta\phi \tag{21}$$

Again, since $b \ll 1/L$, the first term dominates. We thus find the approximate solution

$$\delta\phi \cong \frac{3\bar{\Lambda}L}{4\kappa^2 b\phi_0}\left(e^{4r/L} - e^{-br}\right). \tag{22}$$

To obtain this result, we have assumed that $1/L \gg b, b\phi^2$. (The fact that $b\phi_0$ appears in the denominator of this result might seem to upset our perturbation expansion; we will address this potential problem presently.) The constant of integration was chosen so that the boundary condition $\phi(0) = \phi_0$ is respected by the perturbed solution.

All that remains is for us to impose the two boundary conditions at the second brane. These will determine the new position of this brane, r_Λ, and the 4-D cosmological constant Λ. The b.c. on ϕ is

$$\phi_1 = \phi_0\, e^{-br_\Lambda} + \frac{3\bar{\Lambda}L}{4\kappa^2 b\phi_0}\left(e^{4r_\Lambda/L} - 1\right) \tag{23}$$

and the condition for the brane tension is

$$T_1 = -W(\phi_1)\left(1 + \frac{9\bar{\Lambda}e^{-2A(r_\Lambda)}}{2\kappa^4 W(\phi_1)^2}\right) - \delta W(r_\Lambda) \tag{24}$$

Using the hierarchy $e^{-4A(r_\Lambda)} \gg e^{-2A(r_\Lambda)} \gg 1$, this can be approximately solved for $\bar{\Lambda}$ to give

$$\bar{\Lambda} \equiv \frac{8\pi G}{3}\Lambda \cong \frac{2\kappa^2}{3L}\, e^{-4r_\Lambda/L}(T_1 + W(\phi_1)) \tag{25}$$

This is our main result: if the interbrane distance is around $r_\Lambda \sim \ln(10^{30})L \sim 70L$, then the physical 4-D cosmological constant can be the observed value even if $(T_1 + W(\phi_1))$ is of order the Planck energy density. (Recall that $T_1 = -W(\phi_1)$ is the fine-tuned value that gave us $\Lambda = 0$ in the previous solutions.) To see if such a value of r_Λ is natural, we must solve the ϕ boundary condition (23):

$$e^{-br_\Lambda} = \frac{\phi_1}{\phi_0} - \frac{(T_1 + W(\phi_1))}{2b\phi_0^2} \tag{26}$$

However we must consider the fact that the second term on the right hand side of this equation is of order $1/b$. A small amount of tuning is required so that $(T_1 + W(\phi_1))$ is of order b^2, so that the new term can be treated perturbatively. Once this is done however, there is no great difficulty in achieving the enormous hierarchy of 120 orders of magnitude for Λ. For example if $bL = 0.01$, the r.h.s. of (26) need only be as small as 0.5 to achieve the required separation of $r_\Lambda = 70L$. We have therefore succeeded ameliorating the cosmological constant hierarchy problem in the same way as the original Randall-Sundrum model addressed that

of the weak scale. The mild fine-tuning we demanded for the second brane tension might be merely a technical requirement for the convenience of analytically demonstrating the mechanism. Possibly it still works even without this small amount of tuning, though the solutions are harder to find in that case.

Recently ref. [9] pointed out that any warped compactification solution involving a scalar field must satisfy certain consistency conditions, notably that $\sum_i T_i + \int \phi'^2 dr \cong 0$ in the case where $\Lambda \ll M_p^4$. The present solution has already been shown in [9] to satisfy this relation when $\Lambda = 0$, so we should verify that our perturbed solution also satisfies it. To first order in the perturbation, we should find that $\delta T_1 + 2 \int \phi' \delta \phi' dr = 0$, where δT_1 is the mistuning of T_1 away from its $\Lambda = 0$ value, $\delta T_1 = T_1 + W(\phi_1)$. It is straightforward to show that in the regime where $b \ll 1/L$, $2 \int_0^{r_1} \phi' \delta \phi' dr = \delta W|_0^{r_1} \cong C e^{-4Ar_1}$, and using (20) and (25) that the consistency condition is satisfied.

To complete our argument we should demonstrate that the 4-D Newton constant G can take its observed value without requiring any fine tuning. By integrating over the extra dimension in the 5-D action (2) to obtain the 4-D effective action, one finds the relation

$$\frac{1}{16\pi G} = \frac{1}{4\kappa^2} \int_{-r_\Lambda}^{r_\Lambda} dr \, e^{2A} \cong \frac{L}{4\kappa^2} \tag{27}$$

Thus it is natural to assume that G, κ^2 and L are all of the order M_p to the appropriate power, and no additional tuning is needed to localize gravity.

Although we have succeeded in constructing the approximate solution corresponding to the local minimum of the radion potential, one might wonder why we are not able to find the unstable solution at the local maximum. Evidently this is nonperturbative in Λ. Recall that we needed to do some mild tuning, $T_1 + W(\phi_1) = O(b^2)$ to keep our perturbation expansion under control. Yet the unstable solution exists even when $T_1 + W(\phi_1)$ is exactly zero. It thus seems plausible that the barrier height is large, although we expect in order of magnitude that its size is governed by the same warp factor $e^{-r_\Lambda/L}$ which suppresses the perturbative solution value.
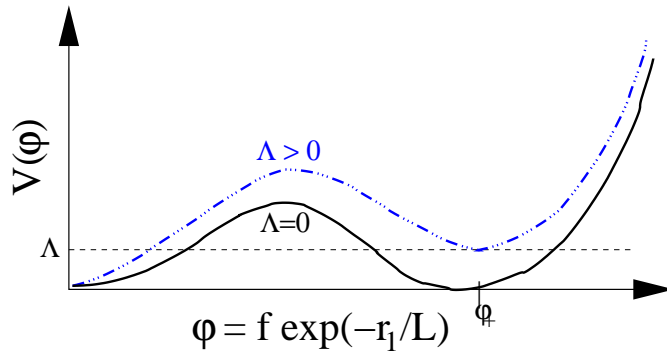

Figure 1: Qualitative form of the radion potential, for the two cases $\Lambda > 0$ and $\Lambda = 0$. The false vacuum state at $\varphi = \varphi_+$ naturally has $\Lambda \sim 10^{-120} M_p^4$.

5. Lifetime of the false vacuum. Our proposal is only viable if the false vacuum has a sufficiently large lifetime that we could still be in it at the present time. To study this we should construct the bounce solution of the Euclideanized radion action and check that

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the lifetime $\tau \sim \Lambda^{-1/4} e^{S_b}$ exceeds the age of the universe, where S_b is the bounce action and $\Lambda^{-1/4} \sim 10^{-4}$ eV is the typical energy scale that will appear in the prefactor in the saddle point approximation for the path integral for the rate of false vacuum decay. We will take the following ansatz for the radion potential:

$$V(\varphi) = \frac{\lambda}{4} \varphi^4 \left[((\varphi/f)^\epsilon - \eta)^2 + (\alpha\eta\epsilon)^2 \right] \tag{28}$$

Note that $\varphi = f e^{-r_1/L}$ is the radion field, not to be confused with the bulk scalar ϕ. In this notation, the small parameter ϵ is related to our superpotential parameters by $\epsilon = bL$, $\eta = \phi_1/\phi_0$, the quartic coupling is $\lambda \cong 4\phi_0^2/(9LM_p^4)$, and $f \cong \sqrt{6}M_p$ [10]. When $\alpha = 0$, this agrees with the approximate form found by ref. [6] in the case where $\Lambda = 0$. It has two degenerate minima at $\varphi = 0$ and $\varphi = f\eta^{1/\epsilon}$, corresponding to the solutions with infinite and finite interbrane distance, respectively, which we found above. (This can be seen through the relation $\varphi/f = e^{-r_1/L}$ between the radion field and the brane separation.) By adding the term $(\alpha\eta\epsilon)^2$, we have lifted the minimum near $\varphi \cong f\eta^{1/\epsilon}$ to a nonzero value of the 4-D cosmological constant:

$$\Lambda = \frac{\lambda}{32} \varphi_+^4 \eta^2 \epsilon^2 \delta \tag{29}$$

where in the limit that $\epsilon \ll 1$, the value of φ at the metastable minimum is given by

$$\varphi_+ \cong f\eta^{1/\epsilon} e^{-\delta/4}; \qquad \delta \equiv 1 - \sqrt{1 - (4\alpha)^2} \tag{30}$$

To estimate the bounce action, we use the thin wall approximation of ref. [11]. For a bubble of radius R in 4-D Euclidean space, with false vacuum energy density Λ,

$$S_b = -\frac{\pi^2}{2} \Lambda R^4 + 2\pi^2 R^3 S_1 \tag{31}$$

where S_1 is the action for the 1-D instanton corresponding to the $\alpha = \Lambda = 0$ limit of the radion potential,

$$S_1 = \int_0^{\varphi_+} d\varphi \sqrt{2V} = \frac{1}{9}\sqrt{\frac{\lambda}{2}} \epsilon\eta(f\eta^{1/\epsilon})^3, \tag{32}$$

and the radius of the bounce solution which minimizes (31) is $R = 3\frac{S_1}{\Lambda}$. Substituting this value into the (4-D) Euclidean action, we obtain

$$S_b \cong \frac{17\pi^2 e^{3\delta}}{\epsilon^2 \eta^2 \lambda \delta^3} \tag{33}$$

which is so large for the parameters of interest for getting a large hierarchy of scales ($\epsilon \sim 0.01$) that the false vacuum state is easily more long-lived than the present age of the universe.

The 1-D instanton obeys the equation of motion $\ddot{\varphi} = \frac{dV}{d\varphi}|_{\alpha=0}$, which can be solved in terms of the Lerch transcendental function Φ,

$$\epsilon\eta\sqrt{\lambda/2}\,t = \frac{1}{\varphi}\Phi\left(\eta^{-1}(\varphi/f)^\epsilon, 1, -1/\epsilon\right), \tag{34}$$

as illustrated in figure 2. (We take $\epsilon = 0.011$ since $\Phi(x, 1, -1/\epsilon)$ is singular when $1/\epsilon$ is a positive integer.) This has the desired behavior that $\varphi \to 0$ as $t \to -\infty$ and $\varphi \to \varphi_+$ as $t \to \infty$.
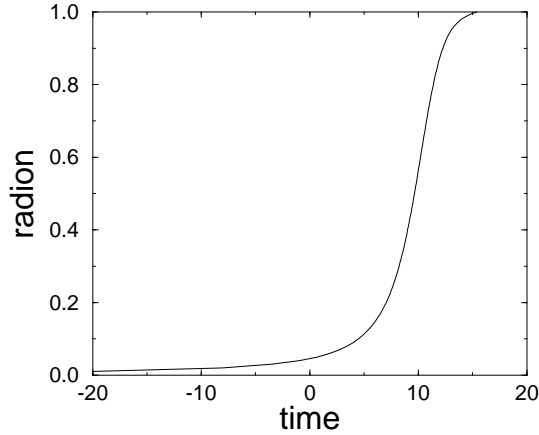

Figure 2: 1-D instanton solution for $\epsilon = 0.011$ and $\delta = 0$. The axes are scaled such that "time" $= \epsilon f \eta^{1+1/\epsilon} \sqrt{\lambda/2}\, t$ and "radion" $= (\varphi/f)\eta^{-1/\epsilon}$.

We can verify that the thin-wall approximation is valid when $\delta \ll 1$. From figure 2, the width is $\Delta R = \Delta t \cong 10(\epsilon f \eta^{1+1/\epsilon} \sqrt{\lambda/2})^{-1}$. Comparing to the bounce radius $R = 3S_1/\Lambda$, we find that $\Delta R/R \cong \delta$.

In ref. [10] we studied a similar problem, that of transitions from a false to a true minimum of the radion potential in the original Goldberger-Wise mechanism. There it was necessary to consider thermally-induced transitions because the radion would have been in thermal equilibrium in the early universe. In the present case, assuming that inflation is driven by an inflaton on the observable brane, the radion is so weakly coupled that it does not come into thermal equilibrium. This is also true of the variant model we propose below in which the observable brane is at the TeV rather than the Planck scale.

6. Physical consequences. In the false vacuum state, the radion will have a very small mass, somewhat below the milli-eV range, since it has been shown that [8]

$$m_r \cong \tfrac{8}{3} b e^{-r_\Lambda/L}. \tag{35}$$

Since $M_p e^{-r_\Lambda/L}$ is supposed to be the meV scale and $b \sim 0.01/L \lesssim 0.01 M_p$, this is a dangerously small mass if the radion were to couple to matter as strongly as does gravity. We have investigated the corrections to this formula due to the $O(\Lambda)$ perturbations and found that they are of the same order as the unperturbed value if $\Lambda \sim b^2$. Hence there is the possibility that corrections to the radion mass which we cannot compute by perturbing in b make it somewhat larger than the value (35).

However, if we live on the positive tension (Planck) brane, the radion's exponentially small couplings to matter make it impossible to observe directly. Let us denote the renormalized trace of the stress energy tensor by $(\tilde{T}^\mu_\mu)_i = e^{-4r_i/L}(T^\mu_\mu)_i$ at the two branes located respectively

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at $r = r_0 = 0$ and $r = r_1 = r_\Lambda$. Because the radion wave function grows like $e^{2r/L}$ away from the Planck brane [12], its coupling to matter on the respective branes goes like [13, 14]

$$\mathcal{L}_{\text{radion}} = \frac{\varphi}{\Lambda^{1/4}} \left((\tilde{T}^\mu_\mu)_1 + e^{-2r_1/L} (\tilde{T}^\mu_\mu)_0 \right).$$ (36)

(In the original RS model, the TeV scale appeared in place of the milli-eV scale $\Lambda^{1/4}$.) If we are living on the Planck brane and $(\tilde{T}^\mu_\mu)_0$ represents standard model physics, then the radion coupling to the standard model is negligible since $e^{-2r_1/L}\Lambda^{-1/4} \sim \Lambda^{-1/4}/M_p^2$.

There is a slightly better chance of detecting the effects of such a radion through cosmology. The shift in size of the extra dimension due to physical energy density ρ on the Planck brane is [8, 13, 15]

$$\frac{\Delta r_1}{r_1} \cong \frac{L\rho}{6r_1 m_r^2 M_p^2}$$ (37)

which becomes of order unity at temperatures $T \sim \sqrt{m_r M_p} \sim 1$ TeV. However this is still such a high temperature that it is difficult to imagine any surviving remnant of the changes to physical scales and the Hubble expansion rate which would arise from a variation in r_1 during this era.

A more testable and interesting situation is to imagine that we are living on a third brane located approximately halfway between the original two, instead of on the positive tension brane. This is desirable apart from considerations of the cosmological constant problem, in that it preserves the natural resolution of the weak scale hierarchy which was the original motivation of Randall and Sundrum. An intermediate brane can be inserted into our solution if it has zero tension and a potential of the form $\gamma_{\frac{1}{2}}(\phi - \phi_{\frac{1}{2}})^2$, with $\gamma_{\frac{1}{2}} \to \infty$ for ease of analysis. The position of this new TeV brane can be adjusted to the desired value by satisfying the approximate boundary condition $e^{-br_{1/2}} \cong \phi_{\frac{1}{2}}/\phi_0$. There are now two radions, one at the TeV scale associated with fluctuations in the distance between the Planck and TeV branes, and the original milli-eV radion.

If there does exist a TeV brane, the coupling of the light radion to particles there will be significantly larger than on the Planck brane:

$$\mathcal{L}_{\substack{\text{radion}- \\ \text{TeV brane}}} \cong \frac{\varphi}{\varphi_+} e^{2(r_{1/2}-r_1)/L} (\tilde{T}^\mu_\mu)_{\frac{1}{2}} \sim \frac{\varphi}{M_p} (\tilde{T}^\mu_\mu)_{\frac{1}{2}},$$ (38)

which means the radion is coupled with approximately the same strength as ordinary gravity. This is precisely the range of scalar masses and couplings which is presently being probed by measurements of the gravitational force at submillimeter range [16]. The new contribution to the potential energy for two masses m_1 and m_2 separated by a distance r is

$$\Delta V = -\frac{4\pi G_N m_1 m_2 e^{-m_r r}}{3r} e^{(4r_{1/2}-2r_1)/L}$$ (39)

The exponential factor should be of order $(\text{meV})^2 M_p^2/(\text{TeV})^4 \sim 6$ for the warp factors to naturally explain the TeV and meV scales of the standard model and cosmological constant, respectively. To evade the constraints of submillimeter gravity tests, this number must be

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made somewhat smaller, or the radion mass must be made somewhat larger than 1 meV. The model is therefore tightly constrained by present tests of the gravitational force.

In addition, the Kaluza-Klein gravitons have a mass gap similar in size to the meV radion, and to the extent that they are nearly massless, their effects will be like those of the KK gravitons in the noncompact Randall-Sundrum scenario [3]. We find that the correction to the Newtonian gravitational potential from these modes is given approximately by

$$\Delta V \cong -G_N L^2 m^2 \frac{m_1 m_2}{r} \frac{e^{-5mr/4}}{1 - e^{-mr}} \left(\frac{1}{4} + \frac{1}{1 - e^{-mr}} \right) \tag{40}$$

where $m \equiv \pi e^{-r_1/L}/L$, and we have approximated the KK masses by $m_n \cong (n + 1/4)m$ using the large-n behavior of the exact eigenvalues [17]. Moreover, the existence of the TeV brane would imply the usual TeV radion, whose phenomenology has been widely studied in connection with the compact Randall-Sundrum scenario [13, 14, 18].

7. Conclusions. We have presented a warped compactification model which, at the expense of assuming there is some unknown solution to the first cosmological constant problem—the question of why the ultimate vacuum energy of the universe is zero—naturally resolves the second one: it explains how the observed value can be 120 orders of magnitude below the Planck energy density without requiring additional fine tuning. The hypothesis is that there exists a brane at such a distance ($\sim 70/M_p$) from the Planck brane that the mistuning of its tension from the flat-brane value contributes to Λ at the $10^{-120}M_p^4$ level, due to the smallness of the warp factor.

Our idea has several shortcomings. Unlike quintessence models, it does not try to solve the coincidence problem of why Λ happens be a significant fraction of the critical density *now*. In the version where we are assumed to live on the Planck brane, we sacrifice any new understanding of the weak scale hierarchy problem, and furthermore there seems to be no experimental signatures that would test the idea. These difficulties are overcome if we live instead on a zero-tension TeV brane between the two original branes, for then the light radion is in the right range for current tests of gravity at submillimeter distances, but then a new fine-tuning problem is introduced: why is the tension exactly (or so nearly) zero? We nevertheless feel that the idea is worth pointing out, in the hope that the problems might be surmounted. For example, ref. [20] has advocated a self-tuning solution [4] to the first cosmological constant problem which avoids the problems of singularities by letting a kink in the scalar field play the role of the Planck brane instead of inserting it by hand. Perhaps such an approach can be used to eliminate some of the tunings that are still present in ours. We find it intriguing that this explanation of the cosmological constant, whose presence is revealed by gravity over cosmological distance scales, might be corroborated by table-top gravity experiments, as well as collider searches.

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